FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2023 FOR RECRUITMENT
TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS = 100**

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Find centre of S_3. (10)

(b) Using the row operations, show that the matrix $\begin{pmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{pmatrix}$ has no inverse. (10) **(20)**

Q. 2. **(a)** For any group G, show that $\frac{G}{\{e\}} \cong G$ and $\frac{G}{G} \cong \{e\}$. (10)

(b) Suppose U and W are distinct four dimensional subspaces of a vector space V of dimension six. Find the possible dimension of $U \cap W$. (10) **(20)**

Q. 3. **(a)** For what value of α is the matrix $\begin{pmatrix} -\alpha & \alpha-1 & \alpha+1 \\ 1 & 2 & 3 \\ 2-\alpha & \alpha+3 & \alpha+7 \end{pmatrix}$ is singular? (10)

(b) Define $T:\Re^3 \to \Re^3$ by $T(x_1, x_2, x_3) = (-x_3, x_1, x_1 + x_3)$. Find $N(T)$. Is T one-to-one? (10) **(20)**

SECTION-B

Q. 4. **(a)** Find the value of θ and the limit in order that $\lim_{x\to 0} \frac{\sin 2x + \theta \sin x}{x^3}$ be finite. (10)

(b) Show that $x < \sin^{-1} x < \frac{x}{\sqrt{1-x^2}}$, 0<x<1. (10) **(20)**

Q. 5. **(a)** Given that $U = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$. Verify that $U_{xx} + U_{yy} + U_{zz} = 0$. (10)

(b) Evaluate $\iint (x^2 + y^2)dxdy$, over the domain bounded by $y = x^2$ and $x = y^2$. (10) **(20)**

Q. 6. **(a)** Evaluate $\iint (x^2 + y^2)dxdy$, over the region bounded by xy=1, y=0, y=x and x=2. (10)

(b) Find an equation of a normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ in the form $ax\cos\theta + by\cot\theta = a^2 + b^2$. Prove that the normal is external bisector of the angle between the focal distances of its foot. (10) **(20)**

SECTION-C

Q. 7. **(a)** Determine k such that $U = e^{2x}\cos ky$ is harmonic and find a conjugate harmonic. (10)

(b) Evaluate $\int_C (\frac{1}{z^5} + z^3)dz$ from 1 to -1 along the upper arc of the unit circle. (10) **(20)**

Q. 8. **(a)** Find the Laurent Series of $\frac{1}{1-z^2}$ in the region $0<|z-1|<2$. (10)

(b) Find the residues at the singular points of $\frac{-Z^2-22z+8}{Z^3-5z^2+4z}$ which lie inside the circle $|z|=2$. (10) **(20)**

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